

Entanglement, thermalisation and stationarity: The computational foundations of quantum mechanics

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"This said, to know others is to be learned, to know oneself, wise - I demonstrate that it could be more fundamental than knowing the rest of nature, by applying classical computational principles and engineering hindsight to derive and explain quantum entanglement, state space formalism and the statistical nature of quantum mechanics. I show that an entangled photon pair is literally no more than a 1-bit hologram, that the quantum state formalism is completely derivable from general considerations of representation of physical information, and that both the probabilistic aspects of quantum theory and the constancy of \hbar are exactly predicted by the thermodynamics of representation, without precluding a fundamental, relative difference in spatial scale between non-colocated observers, leading to logical foundations of relativity and cosmology that show the current thinking in that field to be simplistic and erroneous.

I. MOTIVATION AND OVERVIEW

Everyone knows that "classical theory is *absolutely incapable* of describing the distribution of light from a blackbody" [1, I-41-2] and just about every other result of physics, but the opinion was formed out of frustration a century ago when we did not have an understanding of classical mechanics itself as we do today. I show below how the notions of holographic imaging and computational irreversibility suffice for an elegant classical interpretation of entanglement and the quantum state space formalism, respectively, and conversely, that the quantum postulates have been largely a substitute for this insight. More particularly, the present theory is reversal of the traditional assumption that our knowledge of physics is more fundamental than the science of knowledge itself.

I consider the computational issue of the *observer's physical data states* in the process of observation. By Landauer's principle, these must be irreversibly altered at every observation, unlike in Heisenberg's theory, where the concern was only with possible disturbance to the *observed* entity. I shall also show how quantum uncertainty also results from this perspective, providing a precise computational notion of smallness. The states bear representational correspondence to quantum wavefunctions, yielding, as I shall show, Schrödinger's equation as a computational result, and the constancy of \hbar as a principle of scale in communication between observers.

Of particular concern, naturally, is the probabilistic nature of quantum information, which inspired Schrödinger's cat and many-worlds interpretations, but which, I hope to show with reasonable conviction, is essentially thermal, and in that sense classical. Formal notions of information have always been based on probability arising from thermal motion, beginning with Boltzmann's work in the kinetic theory, but the mere attribution of classical thermalisation does not mean that the underlying *mechanics* should be classical: indeed, the Fermi-Pasta-Ulam (FPU) problem, the difficulty of modelling thermal diffusion by deterministic simulation, already casts doubt on the adequacy of classical mechanisms to guarantee thermalisation. On the other hand, the notion of irreversibility in Landauer's theory plays a key role, as I shall show, in accounting for the probabilistic aspects of quantum theory.

Although I do not attempt to dismiss quantum mechanisms *per se*, for example particle creation and annihilation, the result implies that quantum effects must be indistinguishable from those that would result from thermodynamics and computational considerations anyway. In the one instance where particulate views are weakest, viz radiation in vacuum, the usual attribution of thermalisation to nonlinear interactions becomes logically inapplicable, by passing the FPU issue. I have shown recently [2] that classical considerations of dynamic stationarity under wall jitter, needed to account for the thermalisation classically, exactly reproduce both Planck's law and the harmonic oscillators, which forces me to consider applying classical electromagnetism in other quantum situations as well.

As entanglement is specially of interest in quantum computation, I demonstrate in §II that it is indeed reproduced by classical electromagnetic waves; this is distinct from *all* prior considerations, including those of Bohm [3] [4], and Einstein, Podolsky and Rosen (EPR) [5], as they invariably assumed particles in the classical picture, and were forced into issues of superluminal communication and hidden variables. No such problems occur in the wave picture, classical or otherwise, showing that it is the particulate perception, not classical mechanics, which has been at fault.

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The conclusion is formally supported, as discussed in §III, by a mathematical result by BenDaniel, that representable physical information is necessarily one of continuous fields and not particles. I further demonstrate that the nature of quantum information is fundamentally holographic, which makes entanglement purely an artifact of *a posteriori* selection, consistent with a recent proof by Bennett *et al.* of the impossibility of communicating classical information through entanglement [6].

I then establish my main contention, in §III, that the quantum state space is itself fundamentally computational in origin, by reproducing its complex vector space form purely from considerations of the most general representation for information of any kind, as well as Schrödinger's equation, generalising Dirac's derivation to any such space. This does not suffice to prove the dependence on \hbar , for which I revert, in §IV, to the principles of stationarity and antinodal equipartition already established for the cavity spectrum, and apply these to the classical wave formalism, equivalent to the de Broglie waves of matter, obtainable from Hamilton-Jacobi theory.

Lastly, I show how these principles readily yield the quantisation of matter, quantum uncertainty, and the constancy of \hbar , the last as a transitive result of pairwise interactions among observers and observed entities. In particular, the theory suffices to prove that \hbar is a scale factor relating only the dimensions of energy (mass) [M] and time [T], but not of space [L], permitting a fundamental, relative difference of spatial scale in interactions between separated entities (§V), which directly leads to the logical foundations of relativity and shows the current notions of general relativity and cosmology to be too simplistic to have been correct [7].

II. ENTANGLEMENT: 1-BIT HOLOGRAPHY

Accordingly, I shall first show that classical electromagnetic waves do exactly reproduce quantum entanglement. This generally concerns a bipartite state of two particles of the form $|\psi^\pm\rangle = |a_1b_2\rangle \pm |b_1a_2\rangle$, where the subscripts denote the particles and a and b , their measured properties. Entanglement lies in the inequality of the amplitudes $\langle x_1y_2|\psi^\pm\rangle$, which are probabilistic in quantum theory, but would be deterministic in the classical picture. Key to our result is the observation that the combined bras $\langle x_iy_j|$ can be identified as the resulting combined stationary physical state of two detectors, to be explained in terms of computational principles, and that this state is really a spectral component of the overall physical state, the temporal factor $e^{-i\omega t}$ being implicit in the notation. Also omitted is the fact that the “particles” are travelling; the corresponding factor e^{ikz} is crucial to our classical wave interpretation.

For simplicity of argument, I shall restrict myself to a single Einstein-Podolsky-Rosen (EPR) experiment, in which circularly polarised γ photons are emitted by a disintegrating source and the photons are detected by analysing their linear polarisations. This entails no loss of generality other than that already outlined, viz that the treatment is applicable only to photons, as the resulting arguments will be extensible even to the de Broglie waves of matter. We denote the detection events as $\langle x_1|$, etc., x identifying the linear polarisation and the subscript, the detector; and the initial photon states as $|r_1\rangle$, $|l_2\rangle$, etc., representing the right- and left-circular polarisations, respectively, so that the entangled photon state becomes $|\psi^\pm\rangle = |r_1r_2\rangle \pm |l_1l_2\rangle \equiv |r_1r_2 \pm l_1l_2\rangle$, according to the parity of the parent particle. We would describe the circular polarisation classically by an electric vector field propagating in the z -direction,

$$\mathbf{E}(z, t) = \mathbf{E}_x \cos(kz - \omega t + \alpha) + \mathbf{E}_y \sin(kz - \omega t + \alpha), \quad (1)$$

\mathbf{E}_x and \mathbf{E}_y being the amplitude component in the corresponding directions, so that a detection at z_i , $i \in \{1, 2\}$, means simply that the phase, and therefore the delay t_i , is determined at z_i ,

$$\omega t_i = \alpha + kz_i - \begin{cases} (n + 1/2)\pi \\ n\pi \end{cases}, \quad \alpha \in [0, \pi/4), \quad (2)$$

according to whether the detected polarisation was x or y , respectively, i.e. *the measured bit equivalently determines whether the detector was within an even or odd half wavelength interval from the source*. With one detector, this is all we would have, but with two non-colocated detectors, we could associate the two detections with a common source if, and only if, the ranges were to coincide, or in other words, the two values represented the *same bit* of information. We can reinforce this conclusion by interpreting the quantum kets also classically as

$$\begin{aligned} |r\rangle &\equiv \frac{1}{\sqrt{2}}[|x\rangle + |y\rangle] \approx \frac{1}{\sqrt{2}}[\mathbf{E}_x e^{ikz} + i\mathbf{E}_y e^{ikz}] \\ \text{and } |l\rangle &\equiv \frac{1}{\sqrt{2}}[|x\rangle - |y\rangle] \approx \frac{1}{\sqrt{2}}[\mathbf{E}_x e^{ikz} - i\mathbf{E}_y e^{ikz}], \end{aligned} \quad (3)$$

where \approx denotes the correspondence and $e^{-i\omega t}$ is once again omitted for brevity. The analysed polarisations $\langle x|$, $\langle y|$

likewise correspond to the classical phasors $\mathbf{E}_x^* e^{-ikz}$ and $\mathbf{E}_y^* e^{-ikz}$, respectively, and lead to the classical dot products

$$\begin{aligned}
\langle x_1 y_2 | r_1 r_2 \rangle &\approx \int_{z_1, z_2} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_y^* e^{-ikz_2} \cdot \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_1} + i\mathbf{E}_y e^{ikz_1}] \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_2} + i\mathbf{E}_y e^{ikz_2}] dz_1 dz_2 \\
&= \frac{1}{\sqrt{2}} \int_{z_1} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_x e^{ikz_1} dz_1 \cdot \frac{i}{\sqrt{2}} \int_{z_2} \mathbf{E}_y^* e^{-ikz_2} \mathbf{E}_y e^{ikz_2} dz_2 \\
&= iE^2/2, \quad E \equiv |\mathbf{E}_x| = |\mathbf{E}_y| \\
\langle x_1 x_2 | r_1 r_2 \rangle &\approx \int_{z_1, z_2} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_x^* e^{-ikz_2} \cdot \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_1} + i\mathbf{E}_y e^{ikz_1}] \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_2} + i\mathbf{E}_y e^{ikz_2}] dz_1 dz_2 \\
&= \frac{1}{\sqrt{2}} \int_{z_1} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_x e^{ikz_1} dz_1 \cdot \frac{1}{\sqrt{2}} \int_{z_2} \mathbf{E}_x^* e^{-ikz_2} \mathbf{E}_x e^{ikz_2} dz_2 \\
&= E^2/2,
\end{aligned} \tag{4}$$

since terms involving both z_1 and z_2 in the exponent vanish in the Césaro sum exactly as in quantum theory. Similarly,

$$\begin{aligned}
\langle x_1 y_2 | l_1 l_2 \rangle &\approx \int_{z_1, z_2} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_y^* e^{-ikz_2} \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_1} - i\mathbf{E}_y e^{ikz_1}] \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_2} - i\mathbf{E}_y e^{ikz_2}] dz_1 dz_2 \\
&= \frac{1}{\sqrt{2}} \int_{z_1} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_x e^{ikz_1} dz_1 \cdot \frac{-i}{\sqrt{2}} \int_{z_2} \mathbf{E}_y^* e^{-ikz_2} \mathbf{E}_y e^{ikz_2} dz_2 \\
&= -iE^2/2, \\
\langle x_1 x_2 | l_1 l_2 \rangle &\approx \int_{z_1, z_2} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_x^* e^{-ikz_2} \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_1} - i\mathbf{E}_y e^{ikz_1}] \frac{1}{\sqrt{2}} [\mathbf{E}_x e^{ikz_2} - i\mathbf{E}_y e^{ikz_2}] dz_1 dz_2 \\
&= \frac{1}{\sqrt{2}} \int_{z_1} \mathbf{E}_x^* e^{-ikz_1} \mathbf{E}_x e^{ikz_1} dz_1 \cdot \frac{1}{\sqrt{2}} \int_{z_2} \mathbf{E}_x^* e^{-ikz_2} \mathbf{E}_x e^{ikz_2} dz_2 \\
&= E^2/2,
\end{aligned} \tag{5}$$

yielding the same result as the quantum amplitudes (cf. [1, III-18-3]),

$$\langle x_1 y_2 | \psi^\pm \rangle \approx \begin{cases} 0 \\ iE^2 \end{cases} \quad \text{and} \quad \langle x_1 x_2 | \psi^\pm \rangle \approx \begin{cases} E^2 \\ 0 \end{cases}, \tag{6}$$

the difference being only that the classical amplitudes on the right are exact and not probabilistic. The mystery of entanglement clearly cannot lie in the quantumness of the waves, but only in their treatment as particles [5], which introduces the notion of interaction and communication in the first place. The particulate perception also misses our information argument altogether, viz that entanglement must signify that the detected events concern the same bit of information, representing a common source (eq. 2).

This form of information is well understood in holography, where the image is formed by coincidence of antinodes of multiple waves. In ordinary holography, a single wavelength is used for each colour in the image, and angular diversity, i.e. waves from multiple angles, is used to eliminate aliases in the image by spatial coincidence. One form of holographic radar I worked on [8] employed frequency diversity for the dealiasing: we would have exactly the same principle in our EPR experiment if our source produced photon pairs at multiple frequencies, as eq. (2) would become

$$\omega_j t_{ji} = \alpha + k_j z_i - \begin{cases} (n + 1/2)\pi \\ n\pi \end{cases}, \quad j = 1, 2, \dots \tag{7}$$

With frequency-selective detectors, we would have a pair of detected event bits for each frequency j , but the aliases belonging to any pair of frequencies ω_1 and ω_2 can only coincide within intervals corresponding to the larger of the two wavelengths $\{\lambda_1 \equiv c/\omega_1, \lambda_2 \equiv c/\omega_2\}$, thus diminishing the density of aliases, as well as refining the uncertainty interval to the smaller wavelength. Even with a limited number of query frequencies, it is thus possible to localise the source within a known neighbourhood. In the EPR case, we have only one bit of coincidence information, and thus an infinite number of equally significant aliases, which made the information hard to recognise.

We still need to reexamine the issue of superluminal communication, which implies the conjecture that the detected value at z_2 can be influenced by a measurement at z_1 *after* the emission of the photons from the source. A stronger result, that entanglement as such confers no ability whatsoever to communicate classical information, has in any case already been established recently from within the confines of quantum theory [6], but we need to be able to arrive at

the same result within our imaging interpretation. One difficulty, which could have made our classical wave analysis unthinkable in the past, is that eq. (1) describes a pure, endless sinusoidal wave, not a wave packet with which we could associate a discrete detection event and time-of-flight issues. Even the quantum wavefunctions in the EPR scenario, however, do not denote wave packets at all, but merely individual spectral components, i.e. pure sinusoids, and moreover, we have already established that the multiple spectral components forming a wave packet would only yield a sharper image. Accordingly, we only need to verify that communication *per se* does not occur in the present interpretation either, and the reason may be recalled from past theory concerning relativity, viz that the information represented by the measurement at z_2 is available only *after* correlation with z_1 , signifying *a posteriori* identification.

The EPR correlation is thus mysterious only in the particulate view, which must hence be wrong. Further, although I showed entanglement to be classical only in the case of photons, the imaging principle is identically applicable to any wave formalism, including quantum wavefunctions, meaning that in all such cases, entanglement simply concerns independent detections of a common source. Two other difficulties exist in this regard, first, that a similar classical wave analysis does not appear possible for particles in general, and second, that the quantum wavefunctions and amplitudes are probabilistic, which is again hard to reconcile with the classical mechanics of particles. These are the very ones that make the wave-particle duality conceptually difficult in the first place, as the wave aspect by itself is inherently common to both classical and quantum descriptions, as evidenced above.

Accordingly, I show in the remaining sections that these difficulties too vanish on applying other modern engineering principles from control, computation and thermodynamic theories, the principle in each case being based solely on precise classical reasoning. In §III, I shall first show that the quantum formalism of states is in fact simply the most general representation of information, and therefore the representation necessary and sufficient for analysing the most complex situations including particle physics and the computational principles of the brain. I shall then show, in §IV, how this formalism gets represented and linked to observed physics, in particular, how we arrive at a nonzero \hbar and its constancy, as well as at the probabilistic character and the uncertainty principle of quantum mechanics, also on basis of sound classical, but modern, engineering.

III. REPRESENTABILITY: THE ANTI-PARTICULATE PRINCIPLE

The contention, essentially, is that the quantum formalism of states is foremost computational, and then acquires its probabilistic character once again because of a fundamental principle of computation related to the thermodynamics of representation. The separation was unobvious in the past because the probabilistic nature suggests a closer relation to Shannon's theory, obfuscating the representational aspect of the observer's data states, and yet not revealing the mechanisms that cause it to be statistical in the first place. The distinction is subtle but fundamental, as the logical notion of representation is *per se* deterministic, despite the fact that any physical embodiment is bound to be subject to thermal erosion.

Representability, or definability, as such appears to be sufficient to imply inherent quantisation of fields, as recently shown by BenDaniel [9], the equivalent computational argument being that the total knowledge of any finite set of observers can only be finite to the extent it is representable and communicable, using a finite number of sentences in a language with a finite alphabet, as any other knowledge would be inexpressible by definition and therefore outside the realm of science. Evidently, the premise could have been used avoid many of the measure-theoretic difficulties historically encountered in quantum theory, as suggested in Appendix A, where Cauchy's notion of continuity is also shown to be literally equivalent to our consideration of finite representation. More particularly implied is that:

Physical information fundamentally represents continuous fields and this it can do perfectly; conversely, particles can never be perfectly represented or known, nor directly represented.

One may recall that in classical mechanics, a particle is considered to be a geometrical point requiring no spatial description beyond a triplet of real numbers defining its instantaneous location, with possibly a second triple specifying its velocity. The distinction is that the point coordinates do not suffice to *delimit* the object as a point particle; to delimit without contextual knowledge requires infinite bits of representation. For example, as already explained, with many bits of coincidence information, we can locate a source very closely, albeit still with infinite alias regions, so that contextual knowledge is needed to limit the localisation to one neighbourhood. With infinite information we would be able to locate the source *exactly* and with no contextual knowledge at all, but the exercise assumes that our source is a point entity. No such difficulty occurs if we accept our representative data as inherently referring to a continuous field and not a point particle. This principle formalises our contention in §II that the particulate view is as such erroneous and partly responsible for the semantic difficulties of quantum theory, as will be illustrated again with respect to the notion of intrinsic spin.

To demonstrate the computational origin of the state space formalism, we now attempt to construct the formalism purely by seeking the most general representation for knowledge of any kind. We could pick arbitrary variables, for instance, a combination of groceries, train schedules and stock prices. Despite the generality, we would still have two

conditions available in any such choice, viz numerical values and, if pertaining to real phenomena, observability at various times. In the resulting multidimensional state space, the observed data would be essentially represented in the direction of state vectors, in absence of a meaningful norm for all such spaces. Differences between states could be trivially treated as vectors, and the temporal evolution of a state would be represented only as a continuing change of direction. As shown in Appendix A, we do not need to be able to measure the represented variables with infinite precision at infinitely small intervals; rather, it is our inability in both regards that limits us to finite data of finite precision, and therefore to continuous fields, representing the impossibility of point delimitation, as just explained.

The quantum equations of motion are then obtained as general computational result, by considering the most general form of temporal evolution over such a space. By the representability principle, this too must be limited to a finite number of coefficients of finite precision, the latter being implicit, yielding

$$\sum_{i=0}^n a_n \frac{d^n}{dt^n} |\psi\rangle = |\phi\rangle = F^{(n)}(t) |\psi\rangle, \quad (8)$$

where the kets merely denote our arbitrarily chosen state space, and $F^{(n)}(t)$ signifies a continuously varying applied “force” on the system, assuming our variables are sufficiently well behaved. The generality is the reason we may apply techniques from Laplace transform theory to solve eq. (8); in particular, the operator sum $\sum_{i=0}^n a_n d^n/dt^n$ transforms to $\sum_{i=0}^n a_n s^n$, and yields the characteristic equation $\sum_{i=0}^n a_n s^n = 0$, so that by the fundamental theorem of algebra, the complex plane becomes necessary and sufficient to represent all possible patterns of evolution. We have thus reproduced the broad notion of state space as well as the need for complex valued representation, but without *a priori* assumption of quantisation, so that these ideas are valid for any state space, not just one of quantum theory.

The general state space notion of evolution automatically leads to Schrödinger’s equation, as shown by Dirac [10, §27], because the derivative of the incremental evolution $|\psi(t)\rangle \rightarrow |\psi(t + \Delta t)\rangle$ must have the operator form

$$i\hbar \frac{d}{dt} |\psi\rangle = H(t) |\psi\rangle, \quad (9)$$

where the imaginary coefficient i derives from the fact that vector magnitudes lose significance in state representation, and \hbar is merely a scale factor relating the rate of evolution operator H to the observer’s scale of time. This also leads to spectral decomposition in quantum theory [10, §29], but it should be realised that in general, $e^{i\omega t}$ is simply the imaginary component of (the eigenfunctions of) the Laplace operator $s \sim d/dt$. It is only by restricting to stationary states that the real parts, representing transients $\propto e^{-\alpha t}$, vanish from quantum theory, and the reasons both for stationarity and the constancy of \hbar in eq. (9) are also basically computational, as will be explained in §IV.

The representational freedom explains why we were able to drop e^{ikz} in the quantum versions of eq. (3) and yet arrive at the entanglement prediction, eq. (6); we have paid for our semantic imprecision, of course, by confusing the result with superluminal communication. The quantum notion of spin, as a mysteriously intrinsic property of particles, results from a similar imprecision: if in light of the foregoing theory, we recognise a particle wavefunction $|\psi\rangle \equiv \begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix}$ as a continuous physical field, and remember that we are referring to a travelling wave component, so that both spatial and temporal factors e^{ikz} and $e^{i\omega t}$, respectively, are required for its complete physical representation, it becomes clear that the effect of a Pauli spin matrix, say $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, on $|\psi\rangle$ is to simply rotate the local direction of motion as one travels within the field, literally describing a vortex field rather than an inherently non-geometrical form of angular momentum. It is only in the particulate view, where the complex exponential factors get dropped, that we end up with representational, and consequently semantic, loss. On a related note, in ignoring the magnitudes, we had also lost the E^2 in eq. (6), and this had cost us insight from classical electromagnetism.

IV. THERMODYNAMIC STATIONARITY

I still need to show how \hbar acquires the special significance it does in quantum theory, when applied to mechanical information, and account for the probabilistic nature of quantum wavefunctions, else the foregoing theory would be considered no more than a coincidence.

The first part has been recently achieved [2] by going back to Planck’s theory, and proving that thermal wall jitter, which is in fact necessary to account for the thermalisation of radiation in the first place as there are no nonlinear interactions *between* photons that might suffice for this purpose, unlike the case of gas molecules in kinetic theory; is in fact sufficient to exactly reproduce both Planck’s postulates and the radiation spectrum law. The wall jitter continually changes the stationary modes of the cavity, not just the energy distribution therein; we need to consider

only the fraction of thermal changes that are slower than the electromagnetic transit time across the cavity, because faster motions can only affect transient behaviour. We therefore sum over the equilibrated distribution of modes, and in the process identify families of modes that are related by whole numbers of antinodes. It is trivial to verify that within such a family, the frequencies would be necessarily related as $\{f, 2f, 3f, \dots\}$, and importantly, incremental wall displacements can only change the modal distribution by an integral number of antinodes, i.e. between members of the same family, so that, discounting transients, the harmonic families behave exactly as Planckian oscillators, as each family can have exactly one member energised at any instant. The energy of an antinodal lobe depends only on the amplitude and not the frequency, and in the equilibrated state, the antinodal lobes must have the same energy, reproducing Planck's quantisation rule

$$E = hf; \quad (10)$$

the sums over the families then trivially reproduces the equations in Planck's derivation. The detailed balance, needed for balancing the various contributions, from the thermal Doppler spread due to wall jitter, possible interactions with the wall atoms, nonlinearities due to imperfect vacuum within the cavity, and so on, turns out to be identical in form to the Bose-Einstein derivation. Planck's law

$$\hat{U}(f) = \frac{hf}{e^{hf/k_B T} - 1}, \quad (11)$$

$\hat{U}(f)$ denoting the equilibrated expectation energy at frequency f , thus turns out to be a strictly classical result, stemming from classical electromagnetics and classical thermalisation.

More importantly, the very form of eq. (11) shows that h relates to the antinodal lobes and the spectral domain in almost the same way as the Boltzmann constant k_B does to gas molecules and the spatial domain, as it "exposes" h to measurement the same way as the law of Brownian motion $\langle R^2 \rangle = 6k_B T t / \mu$, R being the mean radial distance travelled as a function of time t , and μ , the mobility; historically exposed k_B , allowing it to be empirically determined from diffusion phenomena [1, I-41-4]. It should also be noticed that an antinodal lobe is indeed the $\lambda/2$ interval of uncertainty we encountered when interpreting entanglement (§II), so that our holographic notion of information would be consistent with a stationary representation in thermodynamic equilibrium, to be explored via Hamilton-Jacobi formalism below.

The constancy of h was also independently established using Dirac's result that given any anti-commuting relation $[\cdot, \cdot]$ and two pairs of conjugate variables u_1, v_1 and u_2, v_2 , we would get, with no assumption of dependence between the pairs, the cross-relation

$$[u_1, v_1](u_2 v_2 - v_2 u_2) = (u_1 v_1 - v_1 u_1)[u_2, v_2] \quad (12)$$

implying that they must have the same constant of anticommutation [10, §21],

$$\begin{aligned} u_1 v_1 - v_1 u_1 &= K[u_1, v_1] \\ \text{and } u_2 v_2 - v_2 u_2 &= K[u_2, v_2]. \end{aligned} \quad (13)$$

We could, for instance, choose classical electric and magnetic wave amplitudes \mathbf{E} and \mathbf{B} within our cavity as u_1 and v_1 , and the induced emf V and current i in a probe for u_2 and v_2 . We should then get the same value for K for the anticommutation of V and i as for E and B . We could next reuse the symbols u_1 and v_1 for V and i , respectively, and identify u_2 and v_2 with the dynamical variables of a third system, establishing the validity of K for the third system, and so on, proving that every physical system that could directly or indirectly interact with our cavity would be similarly quantised with the same value of h , proving it to be a universal constant by transitivity of interaction.

For a general notion of waves and their stationarity in the context of matter, we must, as previously mentioned, turn to Hamilton-Jacobi theory, beginning with the following relation between Hamilton's principal and characteristic functions, S and W respectively, for a single particle system in Cartesian coordinates [11, §10-8],

$$S(q, P, t) = W(q, P) - Et, \quad (14)$$

q being the particle's generalised coordinate, E , its total energy, and P , its momentum, which is a constant of the motion in the configuration space for which S is a solution of the Hamilton-Jacobi equation

$$H(q_1, \dots, q_n; \frac{\partial F_2}{\partial q_1}, \dots, \frac{\partial F_2}{\partial q_n}; t) + \frac{\partial F_2}{\partial t} = 0. \quad (15)$$

W closely relates to the action-angle variables used in the early days of quantum mechanics, the angle variable w being defined as $\partial W / \partial J$, J being the action variable: one would first solve the classical problem using these variables, and

quantisation was achieved by replacing J with a multiple of \hbar . The approach dropped out of vogue for two reasons, first, the classical model was not always solvable and the state space formulation proved simpler and more general, for reasons we have already established, and second, the cause of quantisation was then not known, so that one could not aspire for deeper understanding by sticking to the classical formalism. The second reason is now no longer valid, as the cause of quantisation has been uncovered in the context of radiation equilibrium. It is meaningful, therefore, to reexamine the classical approach in this light, particularly to seek a general classical formalism of travelling waves to which we could apply our antinodal holographic ideas.

Eq. (14) describes constant- S wavefronts travelling in the same direction as the particle: the particle's velocity is given by $u = E/p = E/mv$, and we also have $\mathbf{p} = \nabla W$, but the “waves” are not given to be periodic. We need a spectral decomposition, and since, as Goldstein points out, $S = W - Et$ must be proportional to the phase ωt and W is independent of time, E must be proportional to ω . Since this is similar to Planck's quantisation rule (eq. 10), it was traditionally assumed that classical mechanics stopped just short of quantum theory, as it gave no reason for assuming the constant of proportionality to be nonzero. Indeed, a plane wave trial solution of the form $\psi = \psi_0 e^{iS/\hbar}$, corresponding to $E = \hbar\omega$, would turn Schrödinger's equation (9) to

$$\left[\frac{1}{2m}(\nabla S)^2 + V \right] + \frac{\partial S}{\partial t} = \frac{i\hbar}{2m} \nabla^2 S, \quad (16)$$

via the following well-known form of eq. (9)

$$\frac{\hbar^2}{2m} \nabla^2 \psi - V\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial t} \quad (17)$$

and the derivatives

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar} \psi \frac{\partial S}{\partial t} \quad \text{and} \quad \frac{\partial \psi}{\partial x} = \frac{i}{\hbar} \psi \frac{\partial S}{\partial x} \quad (18)$$

which yield

$$\nabla^2 \psi = \frac{i}{\hbar} \psi \nabla^2 S - \frac{\psi}{\hbar^2} (\nabla S)^2 \quad (19)$$

for the Laplacian of ψ . Eq. (16) would reduce to the Hamilton-Jacobi equation (15) if \hbar were to vanish, and this was the basis for Bohr's Correspondence Principle (BCP), that quantum mechanics reduces to classical theory in the short-wavelength limit, as $\hbar \nabla^2 S \ll (\nabla S)^2$ is equivalent to, for a one-particle system, $(\lambda/p) (dp/dx) \ll 2\pi$.

Observe, however, that for the validity of eq. (14), we could not possibly set $E = 0 \cdot \omega$, i.e. the BCP is not classically meaningful at all. The error lies in mistaking the trial solution $\psi_0 e^{iS/\hbar}$ to be a complete description, instead of as a mere spectral component of the overall dynamics. In the spectral decomposition, we would look for stationary modes for which the Et term in eq. (14) vanishes anyway, and we would not need \hbar to vanish, because the right hand term in eq. (16) then merely refers to a Fourier component, not the whole picture.

Importantly, *stationarity is a necessary condition for both the physical data states of the observer and of the observed entities, as both the observer's knowledge and the represented information cannot change between observations*. This is an essentially computational principle, but it forces us to consider only the stationary states of material systems as being physically relevant. The stationarity further means that the Hamiltonian becomes completely separable (cf. [11, §10-6]), so that the generalised coordinates q_i and momenta p_i are related as in-phase and quadrature components, respectively, at each of the characteristic frequencies. In particular, the combined stationary states of radiation and matter must be stationary with respect to either, and we can then apply Dirac's transitivity argument (eq. 13) to each such pair $\{q_i, p_i\}$ and the $\{\mathbf{E}_i, \mathbf{B}_i\}$ radiation amplitudes at the same frequencies f_i , to infer that the antinodal equipartition must be identically applicable to these stationary modes of matter.

The stationarity and equilibrium principles fully explain both quantisation and the probabilistic aspects of quantum theory. As example, recall that radiation quantisation was conceived to in order to explain the almost instantaneous nature of photoelectric emission [12], but by including the observer's data states in the picture, we can readily see that the abruptness is logically unavoidable, as the observer cannot know of states intermediate to those representable within its own material embodiment. The condition of thermal equilibrium, necessary for potential long term stability of the observer's data states, guarantees, via the spectral decomposition of the Hamilton-Jacobi wave analysis of the observer's embodiment described above, that the data states can only again change in antinodal increments of the same energy as those of radiation. We may therefore represent the observed variables by kets $|\psi\rangle$, the detector values, representing the observer's state space, by bras of the form $\langle\phi|$, and get the same amplitudes $\langle\phi|\psi\rangle$ as when $\langle\phi|$ is set to represent a second observed entity instead, interacting with the first. In either case, the amplitude must be complex

because of the algebraic completeness of the state space formulation. The amplitude is consistent with the (square root of the) probability of the observed system *and* the observer *thermally* and *irreversibly* transiting to the combined state $\langle\phi, \psi\rangle$, the irreversibility being implied by the condition that the $\langle\phi|$ must be itself equilibrated and capable of lasting till erasure or the next observation. As particularly described by Landauer [13], the irreversibility means that the final state is attained regardless of the previous states of the observer and the observed, reproducing the apparent “collapse” of the quantum wavefunction in the process of observation. Lastly, the fact that the transitions can only occur in increments of an antinodal lobe reproduces Heisenberg’s uncertainty principle.

V. SUMMARY AND LOOSE ENDS

To summarise, I have established, though not in this order,

- i. that the quantumstate space formalism is no more than the most general representation of information of any kind whatsoever, which would account for its discovery in the first place, and its applicability in diverse fields ranging from particle physics to studies of the brain (§III);
- ii. that quantum wavefunctions merely constitute Fourier in-phase and quadrature components, as already familiar to electrical engineers working with strictly classical electromagnetism (§III, §IV), and that their quantisation and randomness are due to separate computational and thermodynamic causes (vii, viii, x);
- iii. that the de Broglie waves of matter are obtainable from Hamilton-Jacobi theory, and that the Correspondence Principle was erroneously conceived in the context because the computational requirements of stationarity and thermal equilibrium (viii) were not known;
- iv. that complex values occur in quantum mechanics because of the generality of the information represented, as a result of the fundamental theorem of algebra, which incidentally proves the computational completeness of the quantum formalism that had not been established in prior theory, supporting (i), and relates Schrödinger’s equation to control theory (§III);
- v. that quantum entanglement, as best illustrated by the EPR paradox, amounts simply to 1-bit holography in the classical wave picture (§II), and that issues of hidden variables and action-at-a-distance are merely consequences of preconceived particulate view;
- vi. that the particulate view in fact contradicts formal considerations of the representability of physical information, which show that only continuous fields can at all be physically represented, and thus observed or communicated, without contextual bias, validating the present notions of entanglement (v), Hamilton-Jacobi waves (iii) and the classical field interpretation of spin (§III);
- vii. that both quantisation and the probabilistic nature of quantum wavefunctions are completely explained by the same principles of stationarity and antinodal equipartition (§IV);
- viii. that both stationarity and thermal equilibrium are mandated by the computational consideration of stability of the observer’s data states and the represented knowledge (§III), connecting the representational generality (i, iv, vi) with the actual physics (ii, iii, v, vii);
- ix. that the Hamilton-Jacobi analysis (§IV) establishes not only the sufficiency of stationarity and thermalisation for (vii), but also their necessary involvement in this role, obviating any postulate or alternative explanation; and
- x. that the constancy of h is purely a consequence of the transitivity of these constraints between observed systems as well as between physical observers, signifying the establishment of a universal scale, given that h relates the (independent) dimensions of energy (mass) and time, by communication, again a matter of logic and information.

To illustrate the attribution of quantum probabilities to thermalisation, consider the hypothetical case of an observer frozen to absolute zero temperature. By the third law of thermodynamics (Nernst theorem), such an observer would be frozen into one state of knowledge and would be incapable of making any observations whatsoever. An interesting example of the obfuscation by notions of probability and information theory in the past is the following observation that the germs of BenDaniel’s result were already present in kinetic theory: Boltzmann’s notion of thermodynamic information as the spatial localisation of a gas molecule closely parallels our notion of particle delimitation, and the randomness of molecular motion had nothing directly to do with this measure, as with each bit of information serves to (deterministically) halve the region of uncertainty. The information and randomness aspects were thus separable in

Boltzmann's theory, but were not recognised as such because the determinism properly belongs to the computational issue of representation, and the present distinction of computational and informational aspects does not seem to have been at all made in prior literature.

I have not attempted to discuss how the present theory should be applied to other quantum issues like superconductivity, exchange coupling, *zitterbewegung*, particle creation and annihilation, broken symmetries, or the grand unified theories (GUTs), etc. These are simply too numerous to be recast or corrected by one treatise or author. The fundamental principles of quantum mechanics, which are basis for all such applications, have already been shown to be fundamentally computational and thermodynamic. The fact that the equilibrial antinodal lobe, representing the quantum of change in both radiation and matter waves as described, is independent of the wavelength $\lambda \equiv c/f$ means that the establishment of thermal equilibrium and quantum mechanical consistency do not depend on *a priori* equality of spatial scale between the interacting entities. The hypothesis of relative spatial scale suffices for deducing both the postulates of relativity and Maxwell's equations of electromagnetism, and more importantly, shows that the current ideas in general relativity and cosmology are entirely too simplistic, as separately described in [7]. There again, the problem in prior theory has never been a shortage of mathematical skills, but of the computational intuition of the representability of physical information, and the cognition of fundamental limitations arising from this constraint.

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APPENDIX A: FINITE DOMAIN CALCULUS

The premise of representational finiteness allowed me, in 1983-84, to define a notional framework as follows:

- A. Every function f is represented by at most a finite number of symbols denoting algebraic variables and operations. The continuity of a domain X is likewise defined as the condition that at any finite cardinality $\#(X)$, additional points may be *physically* introduced, *by resizing, adjustment of magnification, or technological replacement, etc.* to indefinitely define new points in the neighbourhood $N(x)$ of every point $x \in X$.
- B. Continuity of a function $f : X \rightarrow Y$ is then defined by the conditions that X and Y are both continuous, every new point $x' \in N_\epsilon(x)$ introduces a computed value $y' = f(x') \in N_\delta(y)$, as in Cauchy's definition.

The point is that Cauchy's definition involves only finite domains at any finite stage in the implied execution of the limit operations, and even the issues of divergence in quantum field theory, for instance, are really concerned with how computed behaviour varies with the precision of the referenced domain, which is implicitly finite in all of mathematics. Like the intuition of continuity given by BenDaniel's result (§III), this is too simple to be obvious.

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